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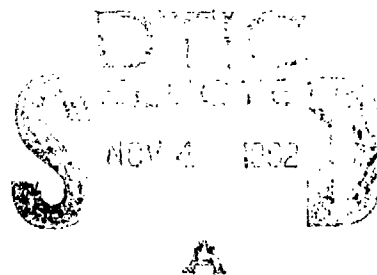
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DIOPHANTINE EQUATIONS AND THE CUBIC AMBIGUITY
RESOLUTION ALGORITHM

TECHNICAL REPORT NO. 84

OCTOBER 1982



MATHEMATICAL SERVICES BRANCH
DATA SCIENCES DIVISION
US ARMY WHITE SANDS MISSILE RANGE
WHITE SANDS MISSILE RANGE, NEW MEXICO



DIOPHANTINE EQUATIONS AND THE CUBIC AMBIGUITY
RESOLUTION ALGORITHM

TECHNICAL REPORT NO. 84

OCTOBER 1982

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SYNCHRONOUS DEDICATED PROCEDURES

Activation

1. Power on equipment.
2. When connected, identify your terminal to the network by entering
D\$\$\$SON XXXXXX (enter your sign-on identification code for XXXXXX)
(Example: D\$\$\$SON AA1234) D\$\$\$SON EMΦ117
3. The system will respond with:

UNIVAC's Telcon Network-Level XXX DCPID=XXX
SESSION PATH OPEN
4. Identify your site to the DROLS system by transmitting the following:
DSGNONS/TERMINAL ID
(Example: DSGNONS/ABCDE) DSGNONS/RTISΦ6
5. The system will respond with:

*MSG ON1 SIGN-ON ACCEPTED
6. Enter your terminal ID as in the past. Entering of the terminal ID at this point will be eliminated in the near future.
7. DROLS commands remain the same.

Termination

1. Enter @TERM@ and transmit.
2. The system will respond with:

THIS TERMINAL HAS BEEN TERMINATED
*MSG D07 PLEASE SIGN OFF TERMINAL
3. Disconnect (sign off) from the network by transmitting:

D\$\$\$SOFF
4. The system will respond with:

SESSION PATH CLOSED
5. Power off equipment.

TERMINAL USER CONDITION MESSAGE DESCRIPTION

TERMINAL MESSAGE	DESCRIPTION	USER ACTION
*MSG ON1 SIGN-ON ACCEPTED	DROLS System has validated the user and allowed access.	N/A
*MSG ON2 SIGN-ON REJECTED	DROLS System error.	Call On-Line Support Office
*MSG ON3 SIGN-ON REJECTED	Sign on error.	Check for data error If correct, call On-Line Support Office
*MSG ON4 SIGN-ON REJECTED	DROLS System error.	Call On-Line Support Office
*MSG ON5 SIGN-ON REJECTED	Terminal is already active.	Continue with DROLS
*MSG ON6 SIGN-ON REJECTED	Illegal Terminal ID used.	Check for data error If correct, call On-Line Support Office
*MSG ON7 SIGN-ON REJECTED	Terminal has been disabled.	Call On-Line Support Office
*MSG OF1 TERMINAL INACTIVE	DROLS System termination error.	Call On-Line Support Office
*MSG DI1 USER TERMINATED	DROLS System error.	Call On-Line Support Office
*MSG DI2 USER TERMINATED	DROLS System error.	Call On-Line Support Office

TERMINAL USER CONDITION MESSAGE DESCRIPTION

TERMINAL MESSAGE	DESCRIPTION	USER ACTION
*MSG D13 USER TERMINATED	DROLS System error.	Call On-Line Support Office
*MSG D14 WAIT-LAST INPUT IGNORED	Terminal is in output mode.	Wait for data to return
*MSG D01 USER TERMINATED	DROLS System error.	Call On-Line Support Office
*MSG D02 USER TERMINATED	DROLS System error.	Call On-Line Support Office
*MSG D03 USER TERMINATED	DROLS System terminating all users.	Standby for Broadcast or call Voice Recorder
*MSG D04 USER TERMINATED	DROLS System terminating this user.	Call On-Line Support Office
*MSG D05 CANNOT INITIALIZE SITE	DROLS System will not allow this site to activate.	Call On-Line Support Office
*MSG D06 USER TERMINATED	DROLS System error.	Call On-Line Support Office
*MSG D07 PLEASE SIGN OFF TERMINAL	Normal termination request.	N/A
*MSG D08 USER TERMINATED	DROLS System error.	Call On-Line Support Office.
*MSG D09 USER TERMINATED	DROLS System error.	Call On-Line Support Office

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A B S T R A C T

The ambiguity resolution problem for the TOAME four phase interferometer system is presented in terms of the integer solutions of systems of linear equations.

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INTRODUCTION

This report deals with data from the TOAME/PDM systems at H-site and A-site. A Diophantine equation approach to ambiguity resolution for four phases is presented. Also the currently used Cubic algorithm is shown to be a variant of this approach. Both algorithms are exercised against difficult data recorded at the sites from a high performance vehicle. Plots of cosine versus time are included. They are self-explanatory. The algorithm presented here performs better than the Cubic algorithm on this data.

A BRIEF REVIEW

Some familiarity with the problem and the nature of the data is assumed. At each site there are two orthogonal base lines. Each base line contains four pairs of antennas with a phase meter for each pair. The distances spanned by each pair are ideally given by the vector $b^T = (43, 42, 36, 288)$ where $1 < \lambda/2$ (slightly less than one-half the wave length of the received signal). Unfortunately the actual spacings of the antennas do not conform too well to these proportions thus introducing modeling errors. The quantities to be ultimately found are the differences in range of the target between the two antennas of each pair. These range differences are measured in cycles of the operating frequency. Designate these range differences by the vector $d^T = (d_1, d_2, d_3, d_4)$. The scaled output of the phase meters p_j , $j = 1, \dots, 4$ is related to the components of d^T by the equations $d_j = n_j + p_j$, $j = 1, \dots, 4$ where the n_j are positive or negative integers and $0 < p_j < 1$. The various algorithms attempt to recover the n_j using the p_j as inputs.

THE DIOPHANTINE APPROACH

Again let $b^T = (43, 42, 36, 288)$ and $d = n + p$ where n and p are vectors consisting of the components n_j and p_j respectively. The planar wave front assumption implies that

$$d = \gamma b$$

or equivalently

$$n + p = \gamma b, \text{ where } \gamma \text{ is a scalar.}$$

Now there is an abundance of 3 by 4 integer matrices A such that

$$Ab = 0$$

Hence

$$A(n + p) = \gamma Ab = 0$$

or

$$An = -Ap$$

The left side of the last equation, An , is clearly an integer vector so it is expected that the right side, $-Ap$, is also an integer vector. If this were true then the system of Diophantine equations

$$An = -Ap = m$$

could be solved for the vector n . However in reality $-Ap$, when computed, is never an integer vector regardless of what suitable matrix A is used. The reasons for this include the facts that the antennas are not quite correctly

positioned, the components of p are stored in finite data registers, and the planar wavefront assumption is not exact. Lump any errors or discrepancies in the vectors b and P into the error vector e . Now rewrite the equation as

$$A_n = -A(p+e) = m + Ae$$

where the error vector e is unknown. About the only thing is to attempt to obtain the integer vector m by means of the nearest integer operator. Denote the nearest integer of x by $ANINT(x)$ in accord with FORTRAN. When the argument is a vector the intent is a component-wise operation. This leads to the Diophantine system

$$A_n = -ANINT(A(p+e)) = ANINT(m + Ae)$$

Even though $ANINT$ is a nonlinear operator, it is true that

$$ANINT(\text{integer} + \text{real number}) = \text{integer} + ANINT(\text{real number})$$

Thus

$$A_n = -ANINT(A(p+e)) = ANINT(Ae) + m$$

If any component of $ANINT(Ae)$ is nonzero then an error is present. Worse yet this error is amplified in the solution of the system of equations. These errors cause spikes and step discontinuities in the cosine plots. Some of the enclosed cosine plots illustrate this phenomenon.

There probably is no certain way of squashing $ANINT(Ae)$ for all conditions, but we can try to minimize the magnitudes of the components of Ae . One way to get a handle on this problem and also state it more elegantly is to say we want to minimize the max-norm (alias ∞ -norm) of Ae . The max-norm of a vector is the absolute value of the component having the largest magnitude. So it becomes apparent that we want a matrix A whose components have small magnitudes or we would like to minimize the induced norm of A . We may not actually minimize this induced norm ($\max_i \sum_j |a_{ij}|$) but we try to make it small and hope for the best.

Start with trial matrix A_0 such that $A_0 b = 0$. For instance let

$$A_0 = \begin{vmatrix} 42 & -43 & 0 & 0 \\ 0 & 6 & -7 & 0 \\ 0 & 0 & 8 & -1 \end{vmatrix}$$

A direct attack on the system $A_0 n = -ANINT(A_0 p)$ encounters awkward difficulties.

It is prudent to factor A_0 in the following way:

$$A_0 = \begin{vmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 0 & 0 & 1 \end{vmatrix} \begin{vmatrix} 1 & 0 & 0 & 0 \\ 0 & 7 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{vmatrix} \begin{vmatrix} 42 & -43 & 0 & 0 \\ -6 & 7 & -1 & 0 \\ 0 & 0 & 8 & -1 \\ 1 & -1 & 0 & 0 \end{vmatrix}$$

Without getting involved in details, the equation $A_0 n = -A_0 p$ can be left multiplied by appropriate matrices resulting in $A_1 n = -A_1 p$ where

$$A_1 = \begin{vmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{vmatrix} \begin{vmatrix} 42 & -43 & 0 & 0 \\ -6 & 7 & -1 & 0 \\ 0 & 0 & 8 & -1 \\ 1 & -1 & 0 & 0 \end{vmatrix}$$

Notice that still $A_1 b = 0$. Furthermore the equation $A_1 n = -ANINT(A_1 p)$ does have an integer solution. However the top row of A_1 contains components of rather large magnitudes. In order to solve this Diophantine system we will disguise somewhat the determination of the extra parameter caused by having 3 equations in 4 unknowns. This is done by an indirect use of the fictitious "very coarse" measurement. Let B_1 designate the righthand 4 by 4 matrix.

That is to say

$$B_1 = \begin{vmatrix} 42 & -43 & 0 & 0 \\ -6 & 7 & -1 & 0 \\ 0 & 0 & 8 & -1 \\ 1 & -1 & 0 & 0 \end{vmatrix}$$

Now let $\bar{m} = -\text{ANINT}(B_1 p)$. Next solve the system $B_1 n = \bar{m}$. The solution is given by $n = B_1^{-1} \bar{m}$. Since $\det(B_1) = 1$ it is guaranteed that B_1^{-1} is an integer matrix and the solution vector n has integer components. It is only the last component of n that is of interest. This is given by

$n_4 = -8(m_1 + m_2) - m_3 + 288 m_4$. The graph entitled 'TEST' shows the plot of $(n_4 + p_4)/(\text{base in feet})$ or \cos against time. This is a real disaster and we suspect the large elements in the first row of A_1 or equivalently B_1 .

Every detail is not being explained here but we are going to use elementary row operations to transform the first three rows of B_1 . We are interested in only those row operations which do not change the magnitude of the determinant. This means we can replace the i^{th} row by minus the i^{th} row and we can replace the i^{th} row by i^{th} row + integer * j^{th} row. We apply these operations with the objective of eliminating large magnitudes among the elements of B_1 .

Specifically make the following simultaneous replacements: 1^{st} row = 1^{st} row + 7 * 2^{nd} row + 3^{rd} row and 2^{nd} row = -1^{st} row - 6 * 2^{nd} row - 3^{rd} row.

This is tantamount to left multiplying B_1 by

$$\begin{vmatrix} 1 & 7 & 1 & 0 \\ -1 & -6 & -1 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{vmatrix}$$

Call the resulting matrix B_2 where

$$B_2 = \begin{vmatrix} 0 & 6 & 1 & -1 \\ -6 & 1 & -2 & 1 \\ 0 & 0 & 8 & -1 \\ 1 & -1 & 0 & 0 \end{vmatrix}$$

Since $\det(B_2) = 1$ it is assured that B_2^{-1} is also an integer matrix. Now let $\bar{m} = -\text{ANINT}(B_2 P)$. Solve $B_2 n = \bar{m}$ for the integer vector $n = B_2^{-1} \bar{m}$. The last component of n is now given by

$$n_4 = 40m_1 + 48m_2 + 7m_3 + 288m_4$$

The graphs of the cosine using this algorithm are entitled 'DIOPH'. It appears to work quite well on the relatively difficult data used here. Each graph of 'DIOPH' is followed by the corresponding graph of the Cubic algorithm. One notable fact is that where the Cubic plot is smooth the quantity $n_4 + p_4$ is bit by bit the same in both algorithms.

The Cubic Algorithm

The Cubic algorithm is currently in use. In order to describe it first define the following two composite inputs.

$p_0 = p_1 - p_2$ which corresponds to a fictitious base line of length unity. This is called the very coarse input.

$p_c = p_2 - p_3$ which corresponds to a fictitious base line of length 6. This is called the coarse input.

Now the algorithm can be described as a three stage procedure.

$$n_c = \text{ANINT} [6p_0 - p_c]$$

$$n_3 = \text{ANINT} [6(n_c + p_c) - p_3]$$

$$n_4 = \text{ANINT} [8(n_3 + p_3) - p_4]$$

The quantities n_3 and n_4 have exactly the same meanings as in the Diophantine solution and n_4 is the quantity of interest. At this point recall that

$$\text{ANINT}(\text{integer} + \text{real number}) = \text{integer} + \text{ANINT}(\text{real number})$$

Apply this to the expression for n_4 .

$$n_4 = 8n_3 + \text{ANINT}(8p_3 - p_4)$$

Substitute for n_3 .

$$n_4 = 48n_c + 8 * \text{ANINT}(6p_c - p_3) + \text{ANINT}(8p_3 - p_4)$$

Continuing.

$$n_4 = 48 * \text{ANINT}(6p_0 - p_c) + 8 * \text{ANINT}(6p_c - p_3) + \text{ANINT}(8p_3 - p_4)$$

Substitute for p_c .

$$n_4 = 48 * \text{ANINT}(6p_0 - p_2 + p_3) + 8 * \text{ANINT}(6p_2 - 7p_3) + \text{ANINT}(8p_3 - p_4)$$

For future convenience rearrange signs.

$$n_4 = -48 * \text{ANINT}(-6p_0 + p_2 - p_3) - 8 * \text{ANINT}(-6p_2 + 7p_3) - \text{ANINT}(-8p_3 + p_4)$$

In the description of a Diophantine equation approach we started with

$$A_0 n = -A_0 p$$

$$n^T = (n_1, n_2, n_3, n_4), p^T = (p_1, p_2, p_3, p_4), A_0 = \begin{vmatrix} 42 & -43 & 0 & 0 \\ 0 & 6 & -7 & 0 \\ 0 & 0 & 8 & -1 \end{vmatrix}$$

Augment this system with the very coarse measurement $p_0 = p_1 - p_2$: Now we have

$$n^T = (n_0, n_1, n_2, n_3, n_4) \text{ and } p^T = (p_0, p_1, p_2, p_3, p_4)$$

The measurement p_0 corresponds to a fictitious base line b_0 where $b_0 = 1$.

Since $1 < \lambda/2$ it follows that always $n_0 = 0$. Furthermore make the redefinitions

$$b^T = (b_0, b_1, b_2, b_3, b_4) \text{ and } A_0 = \begin{vmatrix} 43 & -1 & 0 & 0 & 0 \\ 0 & 42 & -43 & 0 & 0 \\ 0 & 0 & 6 & -7 & 0 \\ 0 & 0 & 0 & 8 & -1 \end{vmatrix}$$

As before

$(n + p) = \gamma b$ and $A_0 b = 0$ hence $A_0 n = -A_0 p$ approximately. This represents four equations in five unknowns. However it is known that $n_0 = 0$ so one of the unknowns is eliminated and we can discard the first column of A_0 to produce

$$B_0 = \begin{vmatrix} -1 & 0 & 0 & 0 \\ 42 & -43 & 0 & 0 \\ 0 & 6 & -7 & 0 \\ 0 & 0 & 8 & -1 \end{vmatrix}$$

and

$$B_0 \bar{n} = -A_0 p, \quad n^{-T} = (n_1, n_2, n_3, n_4)$$

Now we have four equations in four unknowns and B_0^{-1} exists, but unfortunately B_0^{-1} is not an integer matrix. Again start factoring.

$$B_0 = \begin{vmatrix} 1 & 0 & 0 & 0 \\ -42 & 1 & 43 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{vmatrix} \begin{vmatrix} 1 & 0 & 0 & 0 \\ 0 & 301 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{vmatrix} \begin{vmatrix} -1 & 0 & 0 & 0 \\ 0 & -1 & 1 & 0 \\ 0 & 6 & -7 & 0 \\ 0 & 0 & 8 & -1 \end{vmatrix}$$

Next left multiply both sides of $B_0 \bar{n} = -A_0 p$ by

$$\begin{vmatrix} 1 & 0 & 0 & 0 \\ 42/301 & 1/301 & -43/301 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{vmatrix}$$

This is most easily done using the factored expression for B_0 . The result is

$$\begin{vmatrix} -1 & 0 & 0 & 0 \\ 0 & -1 & 1 & 0 \\ 0 & 6 & -7 & 0 \\ 0 & 0 & 8 & -1 \end{vmatrix} \begin{vmatrix} n_1 \\ n_2 \\ n_3 \\ n_4 \end{vmatrix} = \begin{vmatrix} -43 & 1 & 0 & 0 & 0 \\ -6 & 0 & 1 & -1 & 0 \\ 0 & 0 & -6 & 7 & 0 \\ 0 & 0 & 0 & -8 & 1 \end{vmatrix} \begin{vmatrix} p_0 \\ p_1 \\ p_2 \\ p_3 \\ p_4 \end{vmatrix}$$

Again force the lefthand side to have integer components.

$$\begin{vmatrix} -1 & 0 & 0 & 0 \\ 0 & -1 & 1 & 0 \\ 0 & 6 & -7 & 0 \\ 0 & 0 & 8 & -1 \end{vmatrix} \begin{vmatrix} n_1 \\ n_2 \\ n_3 \\ n_4 \end{vmatrix} = \text{ANINT} \left\{ \begin{vmatrix} -43 & 1 & 0 & 0 & 0 \\ -6 & 0 & 1 & -1 & 0 \\ 0 & 0 & -6 & 7 & 0 \\ 0 & 0 & 0 & -8 & 1 \end{vmatrix} \begin{vmatrix} p_0 \\ p_1 \\ p_2 \\ p_3 \\ p_4 \end{vmatrix} \right\}$$

Now the 4 x 4 matrix on the left side does have an integer inverse and the solution to this system of equations is

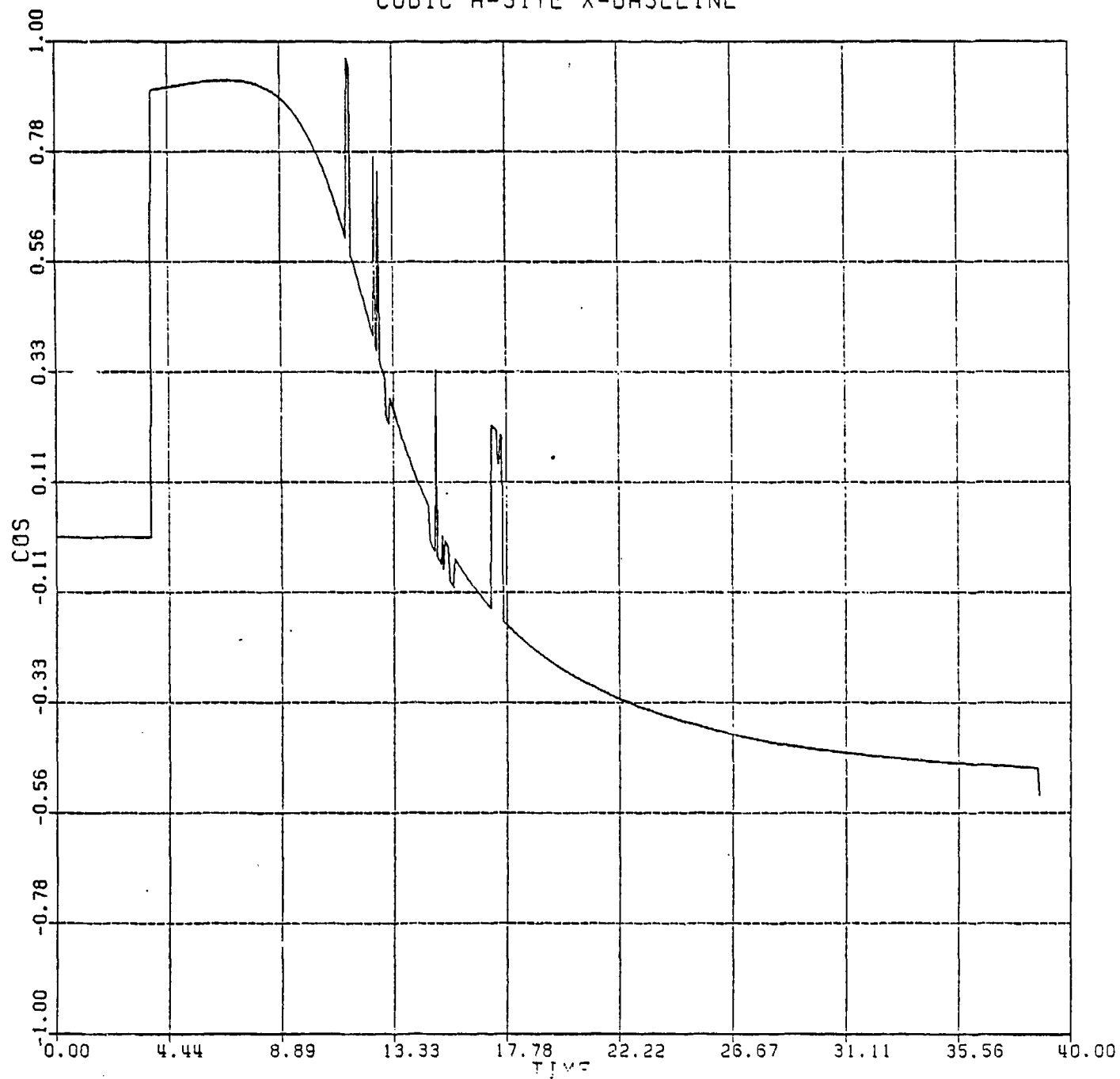
$$\begin{vmatrix} n_1 \\ n_2 \\ n_3 \\ n_4 \end{vmatrix} = \begin{vmatrix} -1 & 0 & 0 & 0 \\ 0 & -7 & -1 & 0 \\ 0 & -6 & -1 & 0 \\ 0 & -48 & -8 & -1 \end{vmatrix} * \text{ANINT} \left\{ \begin{vmatrix} -43 & 1 & 0 & 0 & 0 \\ -6 & 0 & 1 & -1 & 0 \\ 0 & 0 & -6 & 7 & 0 \\ 0 & 0 & 0 & -8 & 1 \end{vmatrix} \begin{vmatrix} p_0 \\ p_1 \\ p_2 \\ p_3 \\ p_4 \end{vmatrix} \right\}$$

In particular

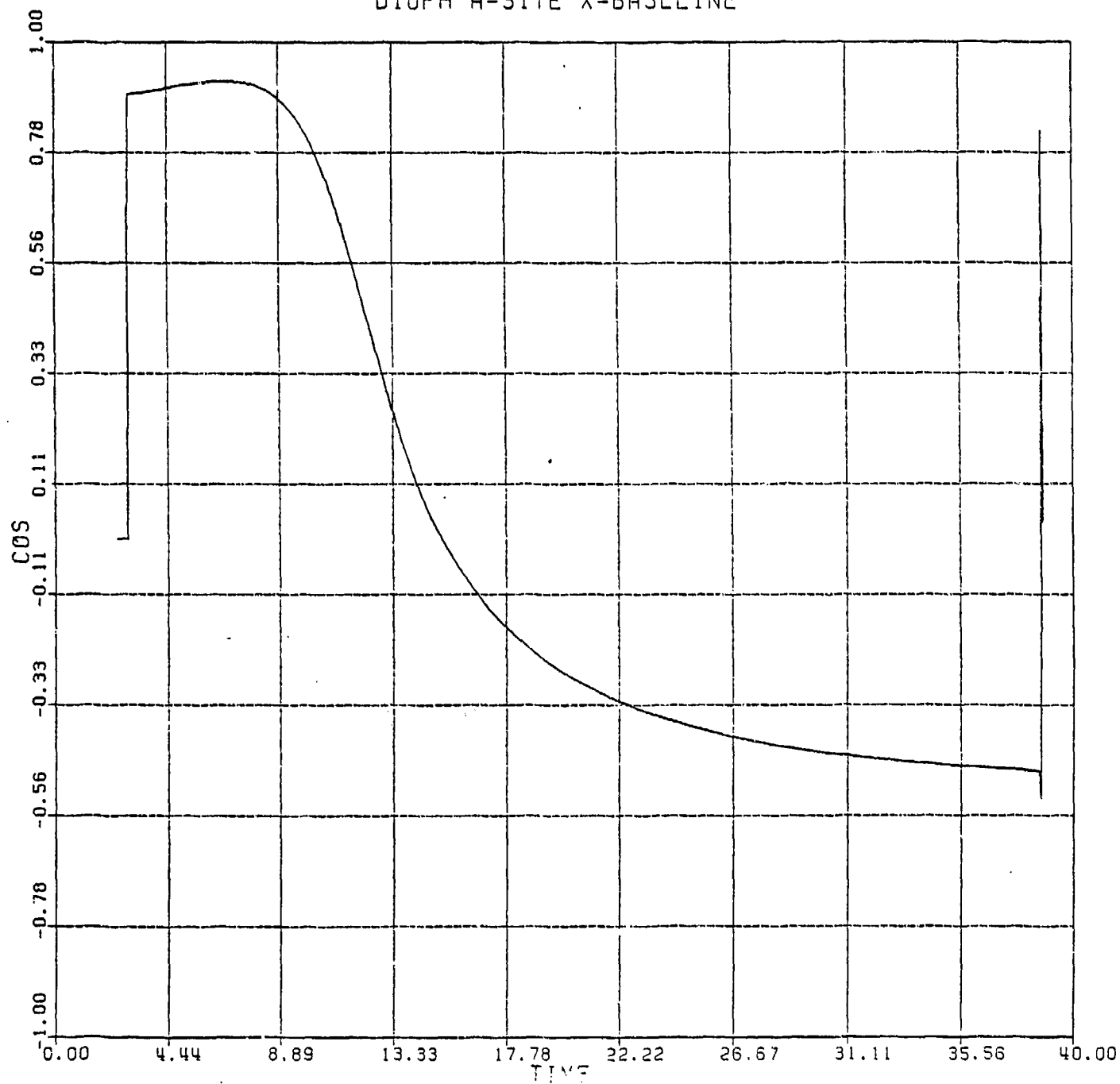
$$n_4 = -48 \text{ANINT}(-6p_0 + p_2 - p_3) - 8 \text{ANINT}(-6p_2 + 7p_3) - \text{ANINT}(-8p_3 + p_4)$$

which is the cubic algorithm.

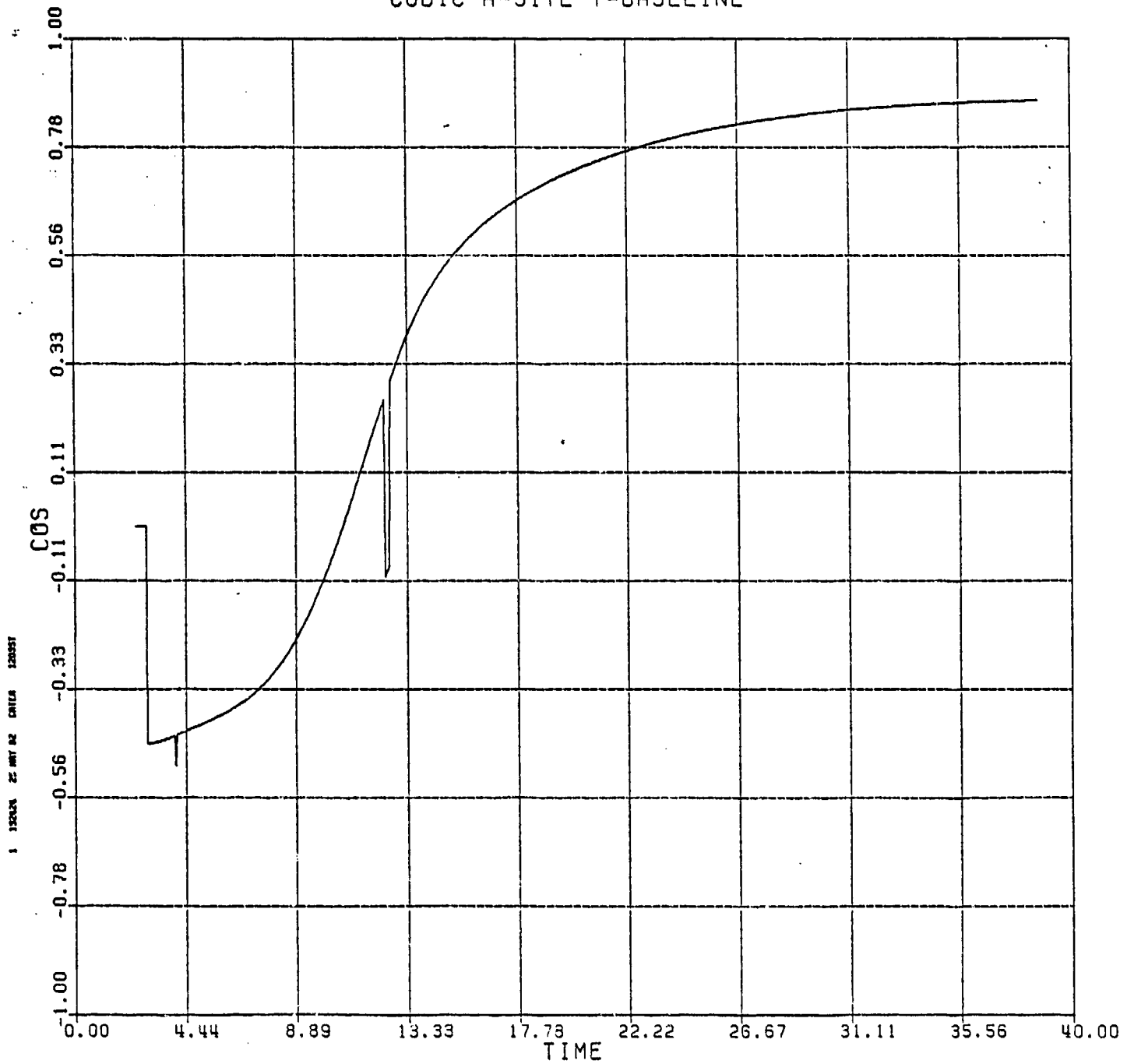
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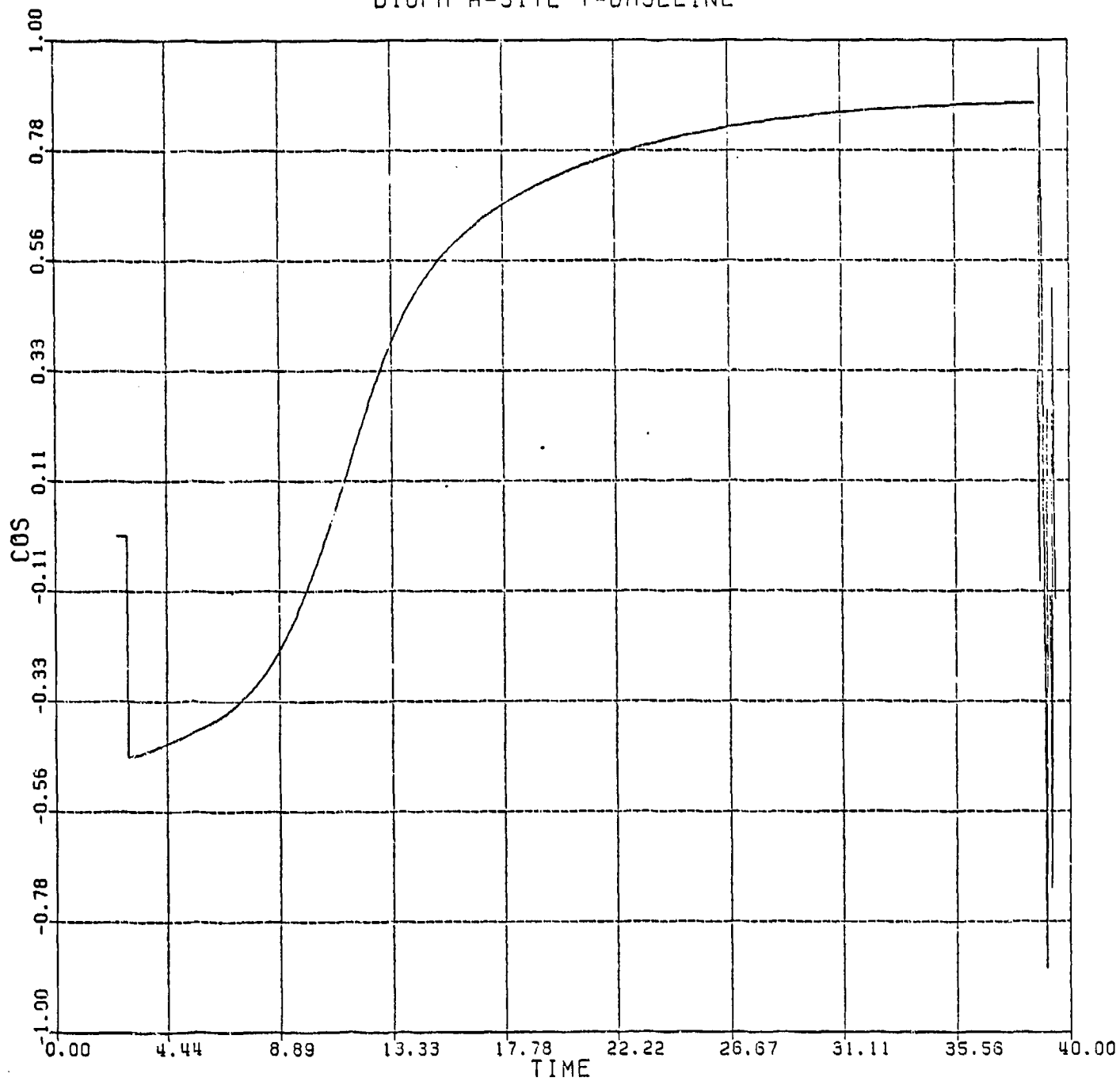
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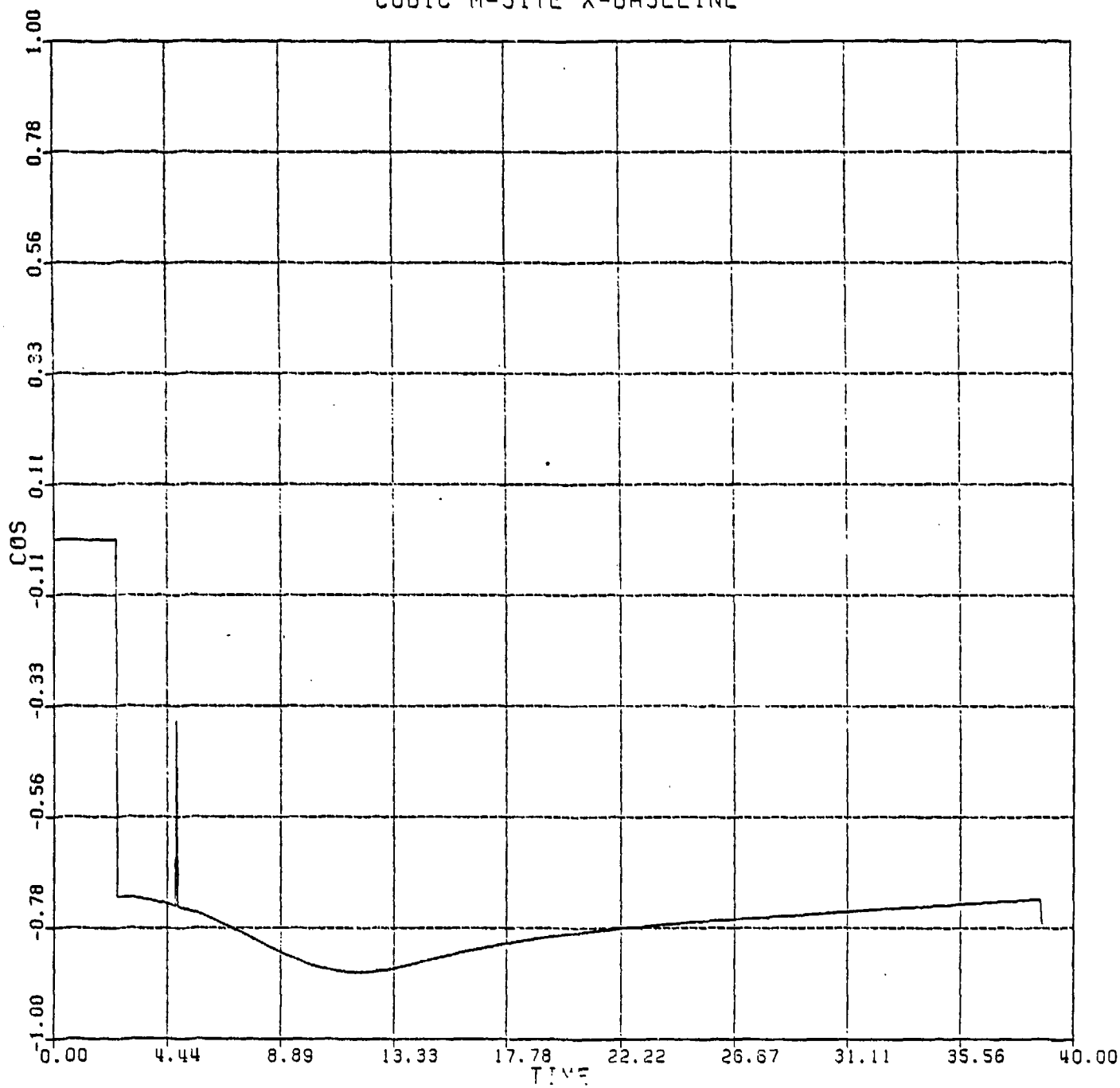
CUBIC A-SITE Y-BASELINE



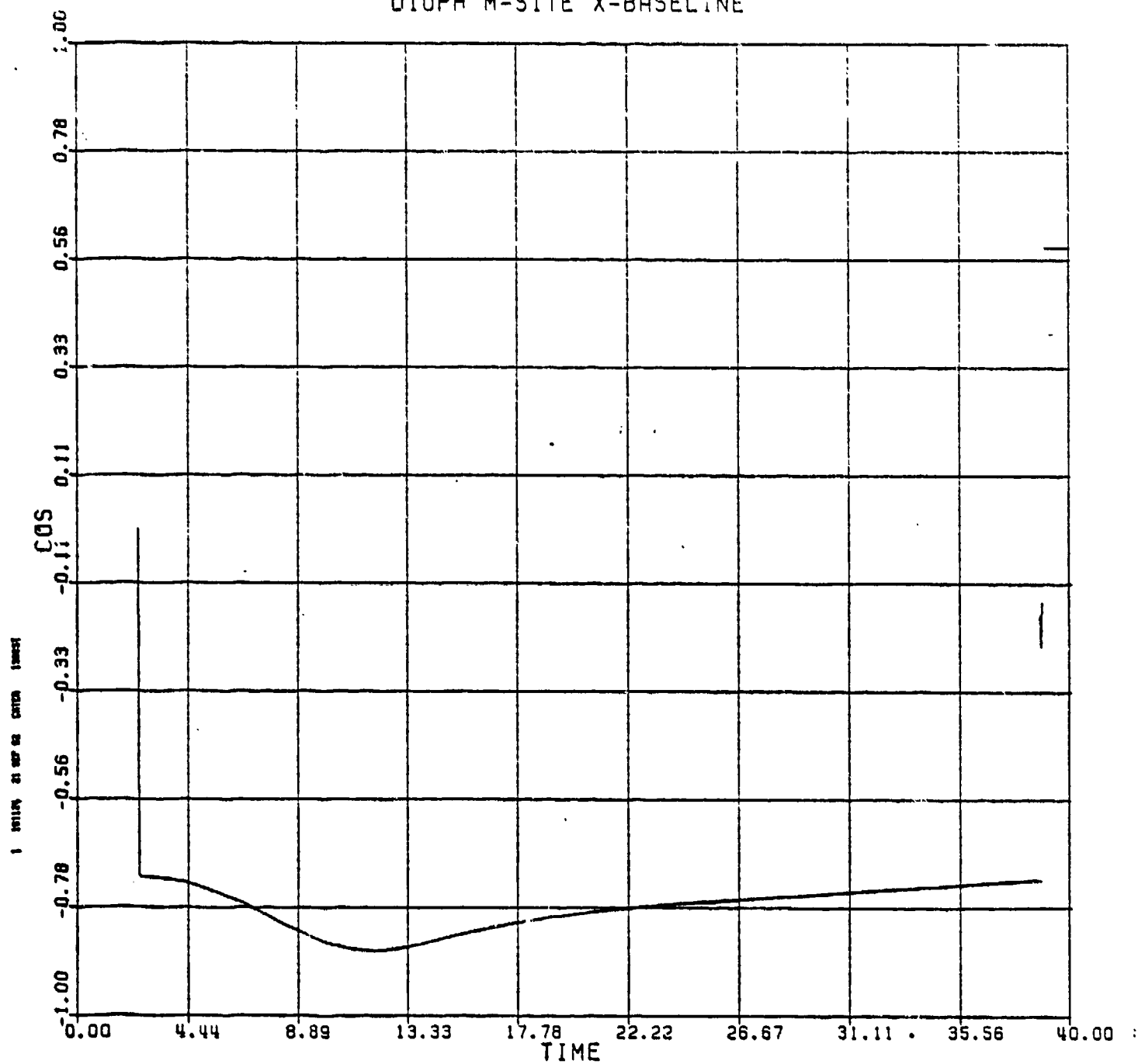
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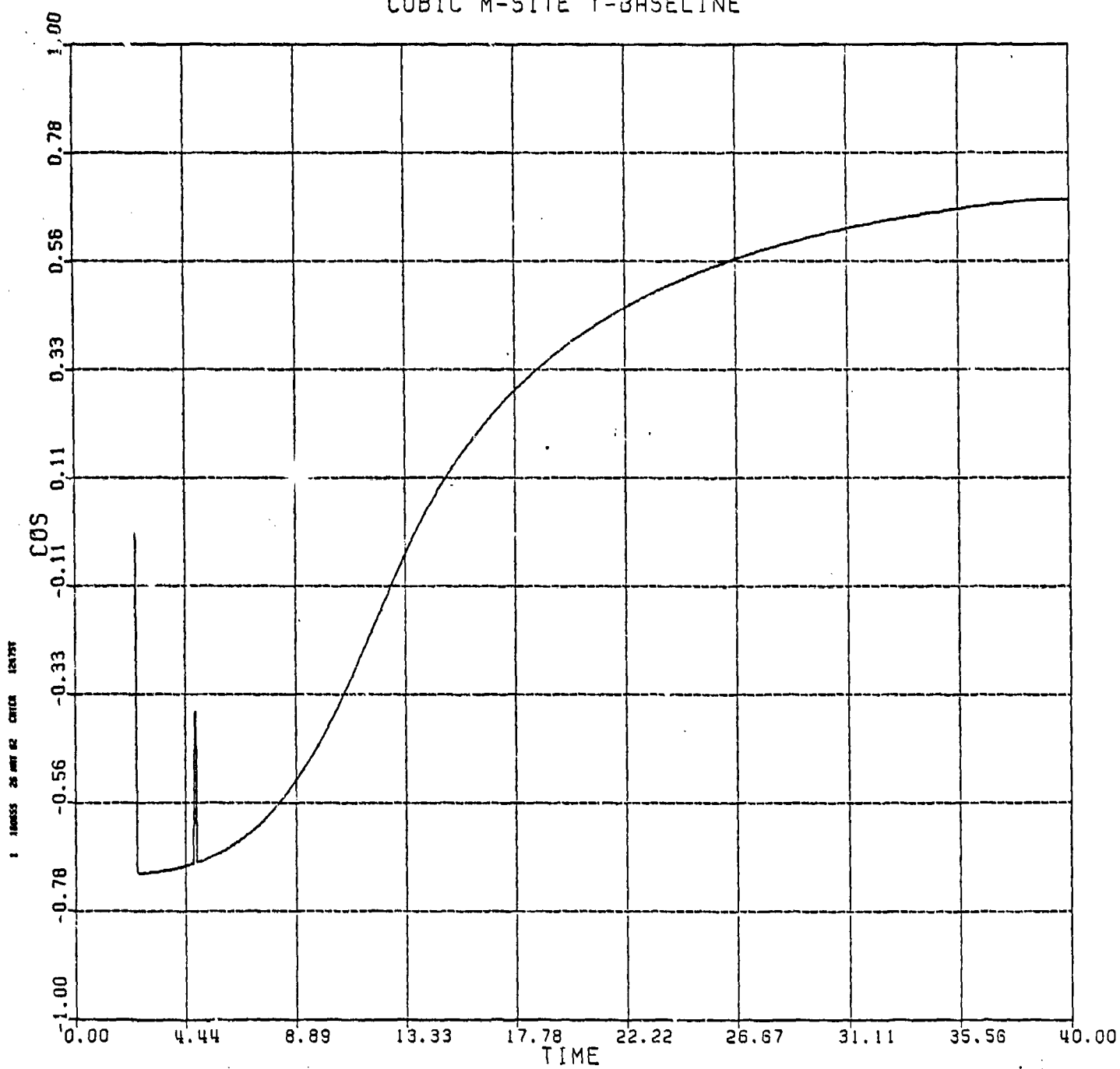
CUBIC M-SITE X-BASELINE



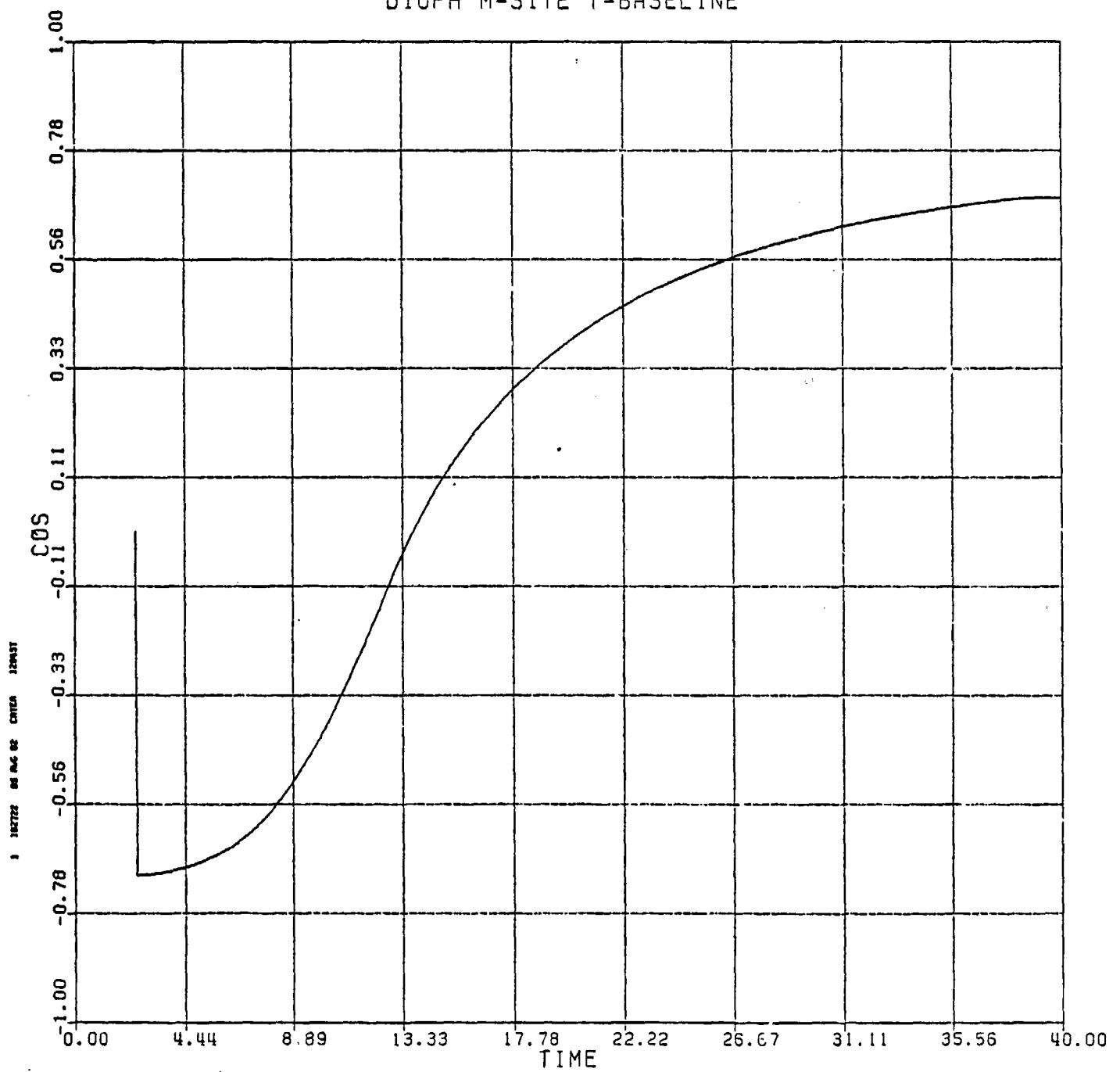
DIOPH M-SITE X-BASELINE



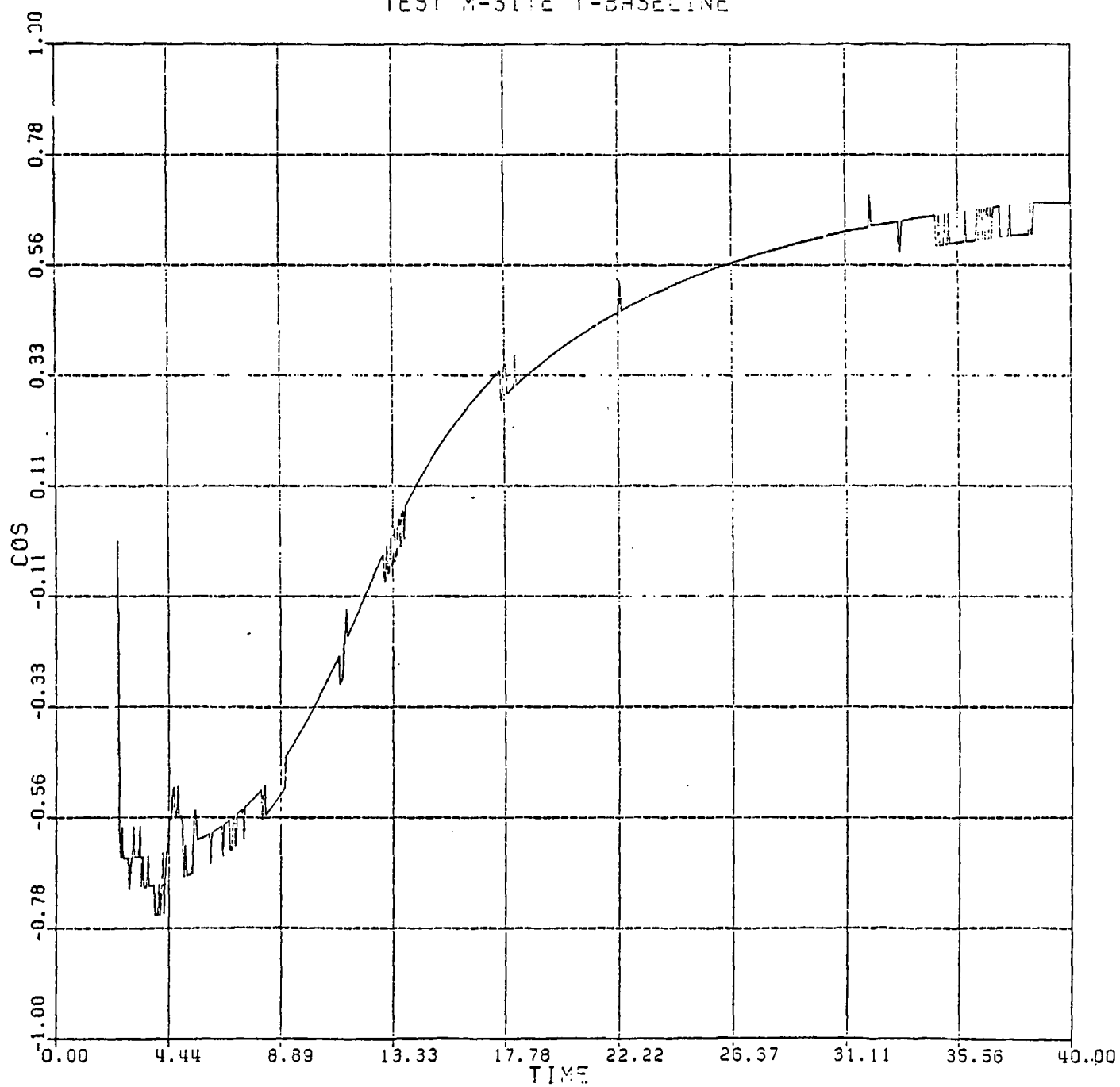
CUBIC M-SITE Y-BASELINE



DIOPH M-SITE Y-BASELINE



TEST M-SITE Y-BASELINE



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